(1)

BIRPASO ARIKETAK

a) $\displaystyle\int \frac{\sqrt{8}\,dx}{6x^2+7} = \sqrt{8}\int \frac{dx}{7\left(\frac{6x^2}{7}+1\right)} = \frac{\sqrt{8}}{7}\int \frac{dx}{\left(\sqrt{\frac{6}{7}}x\right)^2+1}$

$\displaystyle = \frac{\sqrt{8}}{7}\frac{1}{\sqrt{6/7}}\int \frac{\sqrt{6/7}}{\left(\sqrt{\frac{6}{7}}x\right)^2+1} = \frac{\sqrt{8}}{\sqrt{7\cdot 6}}\int \frac{\sqrt{6/7}}{\left(\sqrt{6/7}\,x\right)^2+1} = \boxed{\frac{2}{\sqrt{21}}\operatorname{arctg}\left(\frac{\sqrt{6}x}{\sqrt{7}}\right)+K}$

b) $\displaystyle\int \underset{P}{x^2}\,\underset{S}{\sin x}\,dx = I$ $\left\{\begin{array}{l} u=x^2 \rightarrow du=2x\,dx \\ dv=\sin x\,dx \\ \quad v=\int \sin x\,dx \\ \quad = -\cos x \end{array}\right.$

$\displaystyle I = x^2\cdot(-\cos x) - \int -\cos x\cdot 2x\,dx =$

$\displaystyle = -x^2\cos x + 2\underbrace{\int x\cdot\cos x\,dx}_{I_1}$ $\quad I_2\left\{\begin{array}{l} u=x\rightarrow du=dx \\ dv=\cos x\,dx \rightarrow \\ \quad v=\sin x \end{array}\right.$

$\displaystyle I_1 = x\cdot\sin x - \int \sin x\cdot dx = x\cdot\sin x + \cos x$

$I = -x^2\cos x + 2\left(x\sin x + \cos x\right) + K$

$\boxed{I = -x^2\cos x + 2x\sin x + 2\cos x + K}$

c) $\int \frac{x^2-2x+6}{(x-1)^3}\,dx =$

$$\frac{x^2-2x+6}{(x-1)^3} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^3}.$$

$$x^2-2x+6 = A(x-1)^2 + B(x-1) + C.$$

$x=1 \longrightarrow \quad 5 = \cancel{A0} + \cancel{B0} + C \longrightarrow \boxed{C=5}$

$x=0 \qquad 6 = A + (-B) + 5 \qquad \left| \begin{array}{l} 1 = A-B \\ 4 = 4A-2B \end{array} \right.$

$x=-1 \qquad 9 = 4A-2B+5$

$$- \left| \begin{array}{l} 1 = A-B \longrightarrow \boxed{B=0} \\ 2 = 2A-B \end{array} \right.$$

$$-1 = -A \longrightarrow \boxed{A=1}$$

$$I = \int \left(\frac{1}{x-1} + \frac{5}{(x-1)^3} \right) dx = \ln|x-1| + 5 \cdot \frac{(x-1)^{-3+1}}{-3+1} + k$$

$$\boxed{I = \ln|x-1| - \frac{5}{2(x-1)^2} + k}$$

(3)

d) $\displaystyle\int \frac{\sqrt{x}+1}{\sqrt[5]{x}}\,dx$

ORDEZKAPEN METODOA

$x = t^{10}$ ← errotzaile komuna

$dx = 10t^{9}\,dt$.

$$I = \int \frac{\sqrt{t^{10}}+1}{\sqrt[5]{t^{10}}}\,10t^{9}\,dt =$$

$$= \int \frac{t^{5}+1}{t^{2}}\,10t^{9}\,dt = \int (t^{5}+1)\,10t^{7}\,dt =$$

$$= \int 10t^{12}+10t^{7}\,dt = \frac{10t^{13}}{13} + \frac{10t^{8}}{8} + K =$$

$$\boxed{x = t^{10} \longrightarrow t = x^{1/10} = \sqrt[10]{x}}$$

$$= \frac{10}{13}\sqrt[10]{x^{13}} + \frac{5}{4}\sqrt[10]{x^{8}} + K$$

$$I = \boxed{\frac{10}{13}\,x\,\sqrt[10]{x^{3}} + \frac{5}{4}\sqrt[5]{x^{4}} + K.}$$